Module 6: Physical Processes in Rivers and Lakes

Principles of Surface Water Quality Modeling and Control. R.V. Thomann and J.A. Mueller. Harper & Row, New York. 1987.

Chemical Fate and Transport in the Environment, 2nd edition. H.F. Hemond and E.J. Fechner-Levy. Academic Press. London. 2000.

Physical Processes in Rivers

Fickian Mixing Processes

- A mass of chemical released in a river will spread out as it moves downstream.
- This dispersion is caused by the velocity shear within the river, and turbulent diffusion.
- Water moves more rapidly down the center of the channel, near the surface, transporting chemicals faster, and elongating the "plume."
- Plots of concentration vs. distance has the shape of a Gaussian (normal) curve:

$$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

- For a pulse injection, there is a close relationship between a Fickian mixing, or transport, coefficient D in a given direction and the standard deviation of the chemical distribution in that direction.
- D can be calculated from: $D=\sigma^2/2t$

  where σ^2 is the spatial variance (the square of the standard deviation) and t is the time since the injection.
- The concentration of a conservative tracer (C) at any time (t) after injection and any distance (x) downstream is:

$$C(x,t) = \frac{M}{\sqrt{4\pi D_L t}} e^{-(x-Vt)^2/(4D_L t)}$$

Where D_L is the longitudinal Fickian mixing coefficient $[L^2/T]$

River	Depth (m)	Width (m)	Velocity (m/sec)	Longitudinal dispersion coefficient (m^2/sec)
Irrigation canal	0.14	1.5	0.33	1.9
Monocacy	0.32	35	0.21	4.7
Monocacy	0.45	37	0.32	13.9
Monocacy	0.88	48	0.44	37.2
Yadkin	2.33	70	0.43	111
Yadkin	3.85	72	0.76	260
Susquehanna	1.35	203	0.39	92.9
Sabine	2.04	104	0.58	316
Sabine	4.75	128	0.64	670
Missouri	2.70	200	1.55	1500

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- If the chemical undergoes a first-order decay, then the following predicts downstream concentrations:

$$C(x,t) = \frac{M}{\sqrt{4\pi D_L t}} e^{-(x-Vt)^2/(4D_L t)} \bullet e^{-kt}$$

At any given time t, the maximum concentration of the chemical (C_{max}) is found using:

$$C_{\max} = \frac{M}{\sqrt{4\pi D_L t}} \bullet e^{-kt}$$

- If the chemical is not instantaneously mixed across the river, a “mixing” zone is created. The chemical must travel a certain distance before the chemical is uniform across the channel.
- The lateral standard deviation of the chemical’s concentration distribution can be estimated when this value is approximately equal to the river width (w):

$$\sigma_t = \sqrt{2D_t t} \approx w$$

Substituting the earlier expression for travel time, t, results in the following equation that can be used to predict the length of the transverse mixing zone:

$$L \approx \frac{w^2 V}{2D_t}$$

River type/river	Transverse dispersion coefficients (m^2/sec)	Discharge during dispersion measurement (m^3/sec)
Straight channels		
Atrisco	0.010	7.4
South	0.0047	1.5
Athabasca	0.093	776
Bends		
Missouri	1.1	1900[b]
Beaver	0.043	20.5
Mississippi	0.1	92–120
Meandering		
Missouri	0.12	966
Danube	0.038	1030
Rea	0.0014	0.30
Orinoco	3.1	47,000
MacKenzie	0.67	15,000[b]

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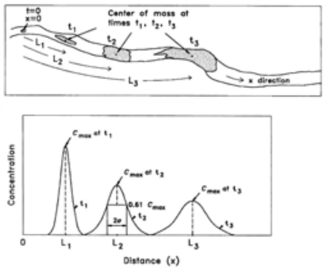
Example Problem 2-2

- The dye concentration profile was measured at time 2 in the dye transport plot, 5 hours after injection. What is the average river velocity if the max. concentration is occurring 1025 m down river from the injection location?



The velocity of the dye from the injection location to the time 2 location is:

$$V = \frac{L_2}{t_2} = \frac{1025m}{5hr} = 205m/hr$$

Estimate the longitudinal dispersion coefficient for this river if the standard deviation, σ_L, in the longitudinal direction is approx. 350 m when the chemical has traveled a distance of 1975 m to L_3

The travel time to this location is:

$$\tau_3 = \frac{L_3}{V} = \frac{1975m}{205m/hr} = 9.6hr$$

The longitudinal dispersion coefficient, D_L, can then be estimated:

$$D_L = \sigma_L^{\ 2}/2\tau = (350m)^2/(2 \bullet 9.6hr) \approx 6400m^2/hr$$

Estimates of Fickian Transport Coefficients from Flow Data

- Turbulence is caused by velocity shear due to a nonuniform velocity profile. The shear velocity (related to the shear force per unit area) can be estimated:

$$u^* = \sqrt{gdS}$$ Where d is the stream depth and S is the slope

This shear velocity can be used to estimate the transverse dispersion coefficient, D_t:

$$D_t \approx 0.15 \bullet d \bullet u^*$$ For straight channels

$$D_t \approx 0.6 \bullet d \bullet u^*$$ For typical natural channels

The following equation can be used to predict the longitudinal dispersion coefficient, D_L:

$$D_L = \frac{0.011 \bullet V^2 \bullet w^2}{d \bullet u^*}$$

Where V is the average velocity [L/T]
w is the stream width [L]
d is the stream depth [L]

Example Design for a Dye Injection Experiment for the Cahaba River

Solve the instantaneous equation for M to determine the amount of conservative dye to be used:

$$C(x,t) = \frac{M}{2A\sqrt{4\pi D_L t}} e^{-(x-Vt)^2/(4D_L t)}$$

1) Estimate the average velocity (V) and travel time (t) to the location of interest (x), and determine the corresponding desired dye concentration (C) at that location.

2) Estimate the longitudinal dispersion coefficient, D_L.

3) Solve for M, the needed mass of dye to be instantaneously released.

1) Estimate the average velocity (V) and travel time (t) to the location of interest (x), and determine the corresponding desired dye concentration (C) at that location.

- The monitoring location is 4 miles from the discharge location (x = 4 miles, 21,120 ft).
- The mean flow for the Cahaba River in this area is 99 MGD, the average width is 30 ft, and the average depth is 1.7 ft.
- The average velocity in this reach is therefore expected to be 3 ft/sec (V=3 ft/sec).
- The travel time is therefore 2 hours (0.08 days)
- The desired dye concentration at the location 4 miles from the discharge location is 250 ppb (v/v).

2) Estimate the longitudinal dispersion coefficient, D_L.

$$D_L = \frac{0.011 \bullet V^2 \bullet w^2}{d \bullet u^*} \qquad u^* = \sqrt{gdS}$$

g = 32.2 ft/sec^2
d = 1.7 ft
S = 0.01
V = 3 ft/sec
w = 30 ft

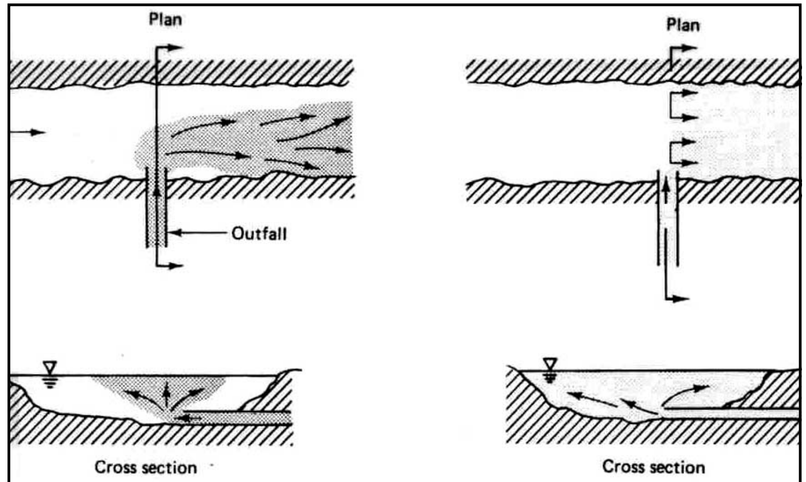
Therefore,
u* = 0.74 ft/sec
D_L = 71 ft^2/sec

3) Solve for M, the needed mass of dye to be instantaneously released.

$$M = \frac{2AC\sqrt{4\pi D_L t}}{e^{-(x-Vt)^2/(4D_L t)}}$$

C = 250 ppb = 250/1,000,000,000 = $2.5\text{x}10^{-7}$
D_L = 71 ft^2/sec
t = 7,040 sec
x = 21,120 ft
V = 3 ft/sec
A = 51 ft^2

Therefore, M = 0.032 ft^3, or 0.24 gal (about 1 L)



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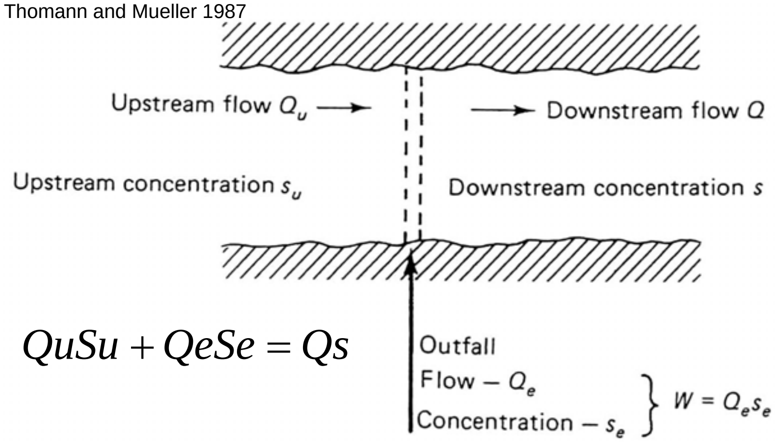

$$QuSu + QeSe = Qs$$

$$S = \frac{QuSu + QeSe}{Q}$$

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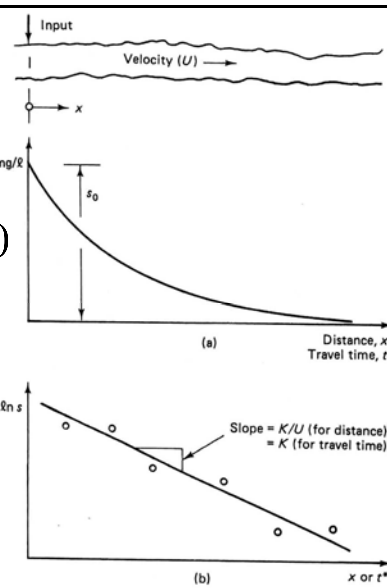
$$S = S_o \exp(-kt)$$

$$S = S_o^{-kt}$$



Example Problem

- Upstream flow is 50 cfs no background concentration of pollutant
- Discharge is 10 MGD at 100 mg/L (k = 0.1/day)
- River velocity is 5 miles/day
- What is the concentration at 10 miles downstream?
- How much reduction is needed if the 10 mi conc. must be < 15 mg/L?

$$Qe = \frac{10x10^6\,gal}{day} \times \frac{ft^3}{7.48gal} \times \frac{day}{86{,}400\sec} = 15.5\,ft^3/\sec$$

$$Q = 50 + 15.5cfs = 65.5cfs$$

$$So = \left(\frac{15.5cfs}{65.5cfs}\right)100mg/L = 23.66mg/L$$

$$S = S_o\exp(-kt) = (23.66mg/L)\exp\left(\frac{-(0.1/day)(10mi)}{5mi/day}\right) = 19.4mg/L$$

$$\left(\frac{19.4-15}{19.4}\right)100 = 23\%reduction$$

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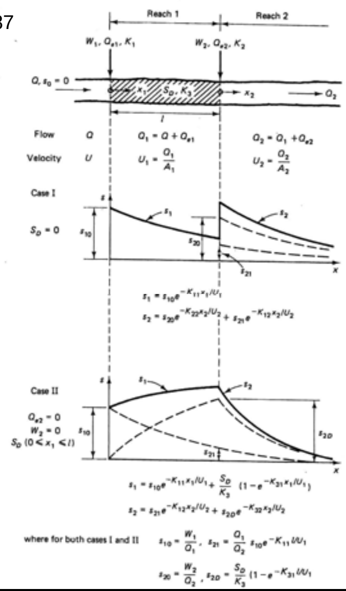

Physical Processes in Lakes

Fickian Mixing in Lakes

- A mass of tracer injected into a lake will move by advection with the water currents, but will also spread out into an ever-larger volume of water.
- Given enough time, it will tend to become completely mixed.
- This mixing is primarily due to turbulence, carrying chemicals away from regions of higher concentrations to areas of lower concentrations.

- Concentrations for an instantaneous discharge into a two-dimensional body of water (vertically mixed):

$$C(x,y,t) = \frac{M}{4\pi t\sqrt{D_x D_y}} e^{-((x-V_x t)^2/(4D_x t)+(y-V_y t)^2/4D_y t)} \bullet e^{-kt}$$

M is the mass of the chemical discharged, per depth of water [M/L]
x and y are the distances from the injection location [L]
t is the time lapsed since injection [T]
V_x and V_y are the average velocity in the x and y directions [L/T]
D_x and D_y are the Fickian transport coefficients in the x and y directions [L^2/T]
K is the first-order decay rate constant [T^{-1}]

The depth is the total depth for a vertically well-mixed lake, or the thickness of a layer in a stratified lake.

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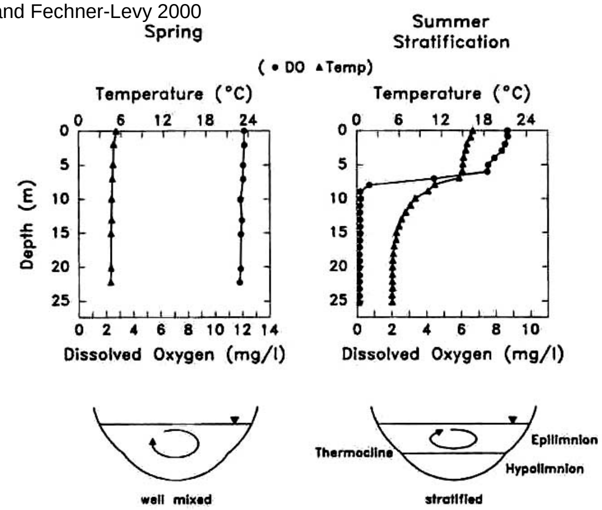
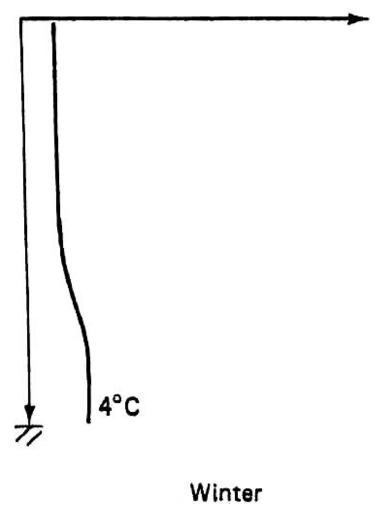


What would lake temperatures look like in winter (very cold area)?



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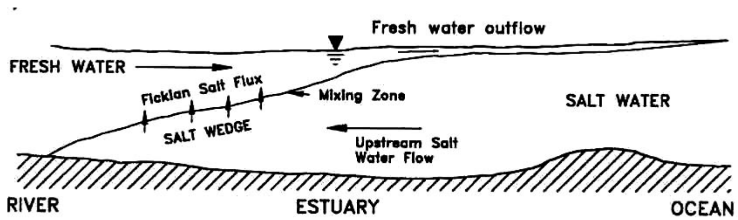

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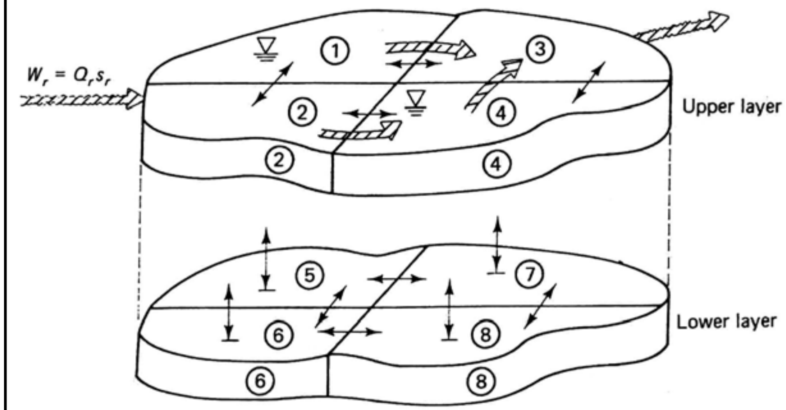

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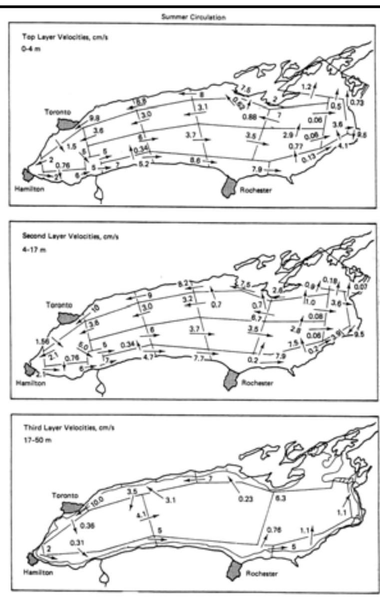

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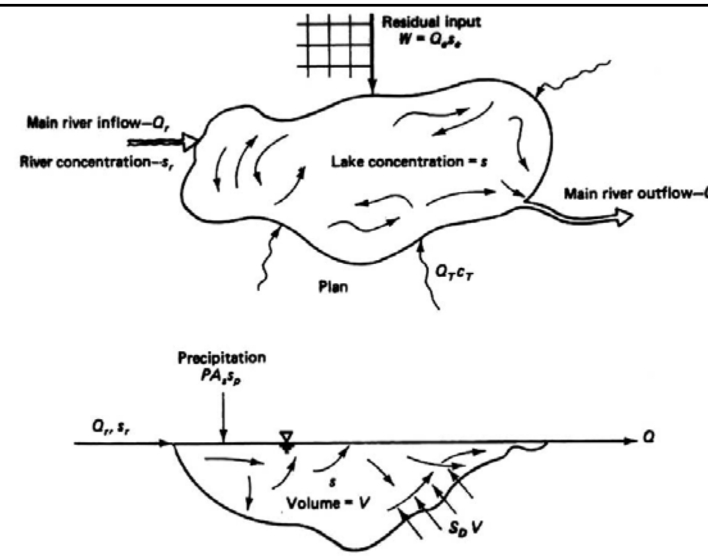

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$$W = QeSe + QrSr + Q_T S_T + PAsSp + S_D V$$

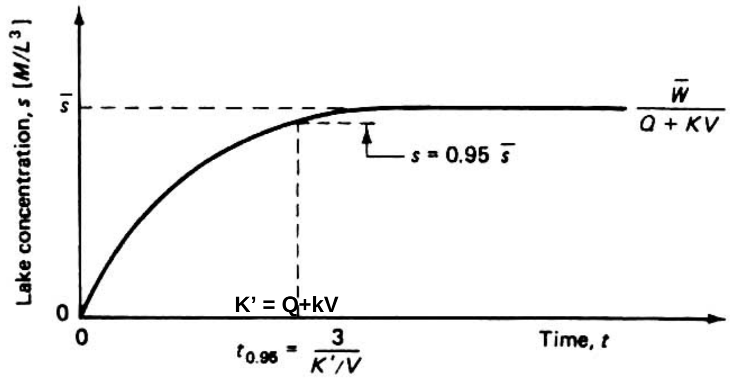
W= mass input [M/T]
QeSe = waste effluent discharged to lake
QrSr = mass from main river
QtSt = mass from tributary
PAsSp = mass input from precipitation
SdV = sediment release

Qe = effluent discharge
Qr = river flow
Qt = tributary flow

P = precipitation amount
As = lake surface area
V = lake volume

Se = effluent concentration
Sr = river concentration
St = tributary concentration
Sp = rain concentration
Sd = sediment concentration

Concentration increases with time, since the start of the discharge:



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Concentration Decrease after Discharge Stops

$$\frac{dVs}{dt} = W(t) - Qs - kVs$$

Change of mass with time =
input mass (gain) – mass outflow (loss) – decay (loss)

Assuming a constant Q and k over time

Expanding the derivative:

$$\frac{dVs}{dt} = V\frac{ds}{dt} + s\frac{dV}{dt}$$

If V is temporarily constant: $\frac{dV}{dt} = 0$

Then: $$\frac{dVs}{dt} = V\frac{ds}{dt}$$

Re-arranging results in:

$$W(t) = V\frac{ds}{dt} + Qs + kVs$$

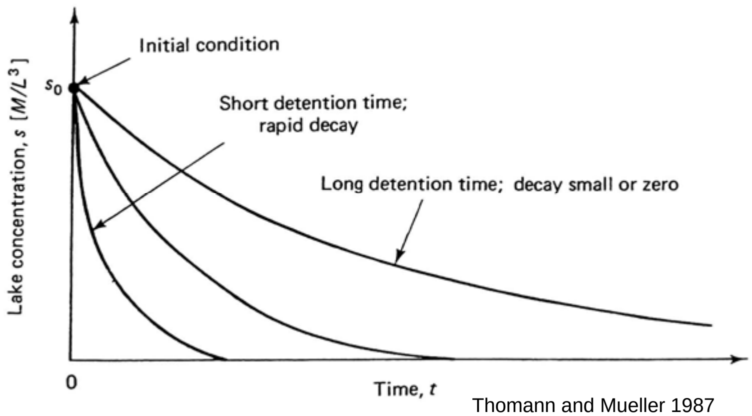
Can simplify if define: $k' = Q + kV$

Then: $$W(t) = V\frac{ds}{dt} + k's$$

Resulting in: $$s = s_0 \exp\left[-\left(\frac{1}{t_d} + k\right)t\right]$$

Where the lake detention time, td is defined as: $t_d = \frac{V}{Q}$

Concentration decreases from initial conditions (at end of input) by a combination of flushing and decay:



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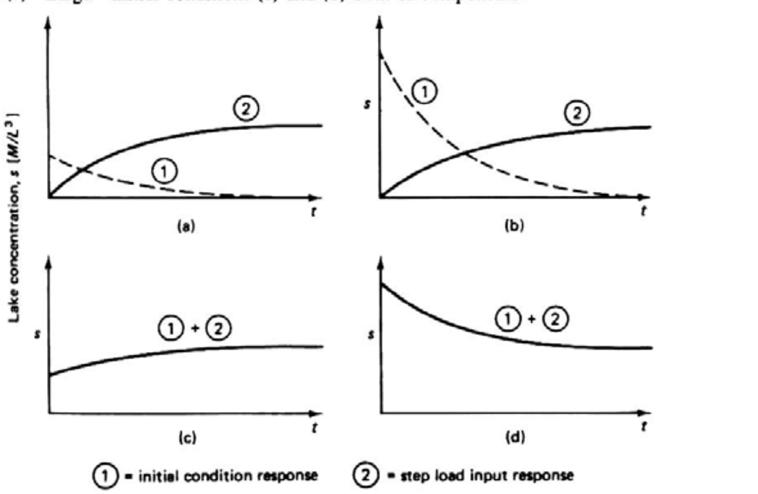
Response due to step load = sum of initial condition (from previous load), plus step load effect:

$$s = \frac{W}{Q + kV}\left\{1 - \exp\left[-\left(\frac{Q}{V} + k\right)t\right]\right\} + s_o \exp\left[-\left(\frac{Q}{V} + k\right)t\right]$$

Total response and transitions can be determined by calculating individual responses and summing the effect.

Response due to initial condition and step input. (a) "Small" initial condition. (b) "Large" initial condition. (c) and (d) Sum of components.



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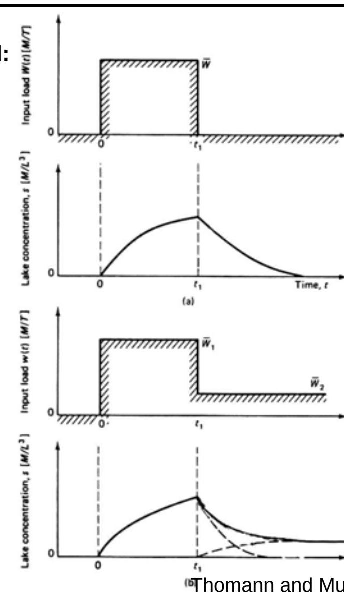
Response due to varying load:

(a) Step input and subsequent reduction to zero

(b) Step input with reduction to new load level



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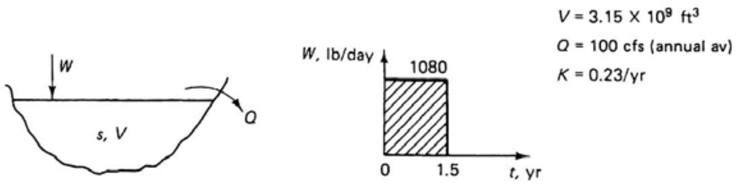
Example Problem:

A Lake (with initial s = 0), receives a load of a slowing reacting pesticide (triallate) of 1080 /b/day for 1.5 years and is then terminated.

1. Determine the equilibrium concentration
2. The maximum concentration
3. The time until a level of 100 μg/L is reached



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(1) Determine the equilibrium concentration

$$\bar{s} = \frac{W}{Q + kV} = \frac{W/Q}{1 + kt_d}$$

$$t_d = \frac{V}{Q} = \frac{3.15x10^9\ ft^3}{100\ ft^3/\sec} x \frac{day}{86{,}400\sec} x \frac{year}{365 days} = 1.0 year$$

$$\bar{s} = \frac{W/Q}{1 + kt_d} = \frac{\dfrac{1080 lb/day}{100\ ft^3/\sec x 86{,}400 \sec/day}}{1 + \left(\dfrac{0.23}{day}\right)(1.0 year)} = 0.000102 lb/ft^3$$

$$\bar{s} = \frac{0.000102 lb}{ft^3} x \frac{ft^3}{7.48 gal} x \frac{gal}{3.78L} x \frac{454{,}000mg}{lb} = 1.63 mg/L = 1630 \mu g/L$$

(2) Determine the maximum concentration

(the max. concentration will occur at the end of the discharge time, at t=1.5 years)

$$s = \bar{s}\left\{1 - \exp\left[-(1 + kt_d)\left(\frac{t}{t_d}\right)\right]\right\}$$

$$s = 1630 \mu g/L\left\{1 - \exp\left[-\left(1 + \frac{0.23}{yr}1.0yr\right)\left(\frac{1.5yr}{1.0yr}\right)\right]\right\} = 1370 \mu g/L$$

Never reaches the equilibrium concentration before it starts to decrease.

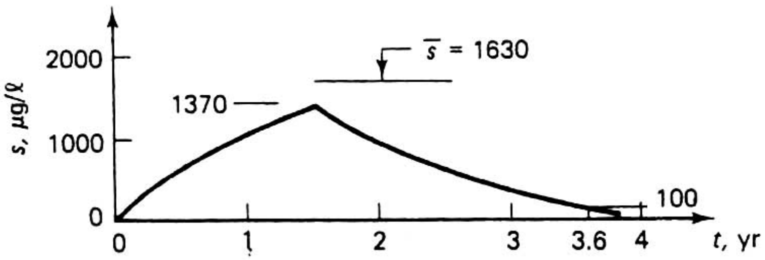
(3) Determine the time until a level of 100 μg/L is reached

$$s = s_0 \exp\left[-(1 + Kt_d)\left(\frac{t'}{t_d}\right)\right]$$

Where: $t' = t - 1.5 years$

$$100 \mu g/L = 1370 \mu g/L \exp\left[-\left(1 + \frac{0.23}{yr}1.0yr\right)\left(\frac{t'}{1.0yr}\right)\right]$$

$$t' = 2.13 years$$



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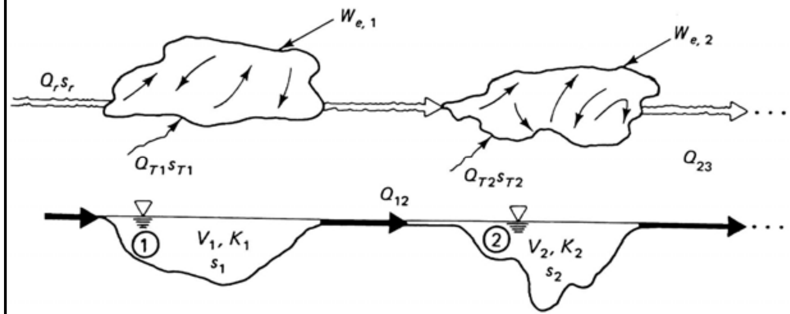

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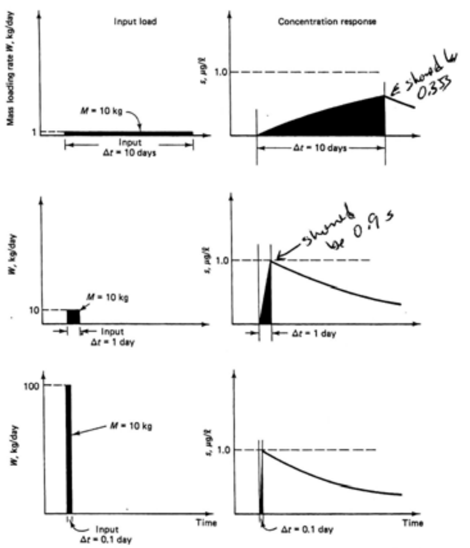


Illustration of response due to an input load discharged over various Δt time intervals ($1/t_d + K = 0.10$/day; $V = 10^7$ m^3). Thomann and Mueller 1987

Movement of Particulates in Natural Water Bodies

Stoke's Law to predict settling of particles (type 1, discrete, non-interacting):

$$\omega_f = \frac{(2/9) \bullet g \bullet (\rho_s / \rho_f - 1) \bullet r^2}{\eta_f}$$

With the sediment and fluid densities, particle size, and kinematic viscosity.

Downward flux due to settling [M/L^2T] is therefore:

$$J_{Stokes} = C \bullet \omega_f$$

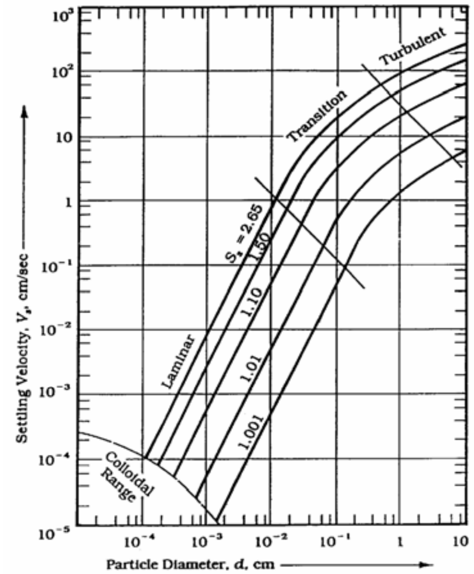
Corresponding upward flux density:

$$J_{Fickian} = D \bullet dC / dx$$

Vertical concentration profile of particles at steady state:

$$C = C_o \bullet e^{-(\omega_f / D) \bullet x}$$

Particle Settling Rates; Stoke's and Newton's Laws



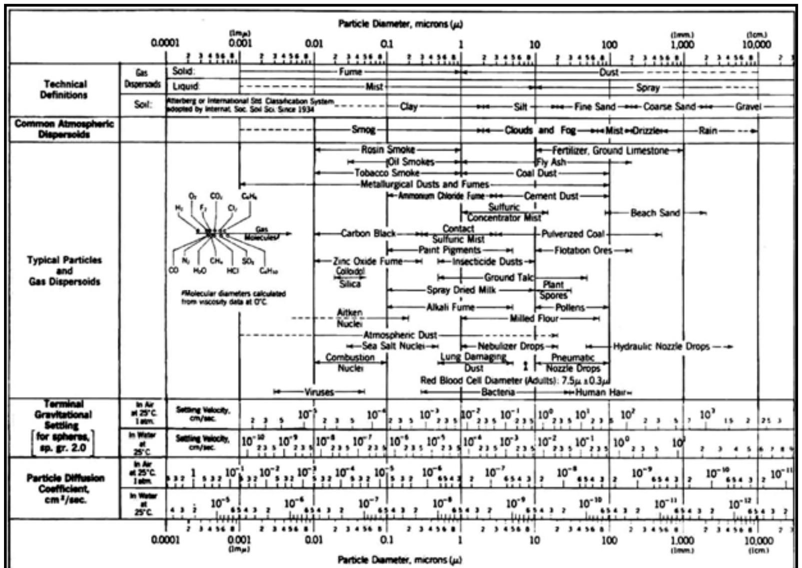
Example 2-3

Describe the steady-state distribution of 1 μm clay particles in still water. Assume kinematic viscosity of water at 50ºF is 0.013 cm^2/sec and a solid density of 2.6 g/cm^3. Also calculate the depth when the concentration is halved.

The settling rate for the 1 μm particle is therefore:

$$\omega_f = \frac{(2/9) \bullet g \bullet (\rho_s / \rho_f - 1) \bullet r^2}{\eta_f} = \frac{(2/9) \bullet 981 cm/\sec^2 \bullet ((2.6 g/cm^3)/(1 g/cm^3) - 1) \bullet (5X10^{-5} cm)^2}{0.013 cm^2/\sec}$$

$$= 6.7X10^{-5} cm/\sec$$



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The diffusion coefficient for this particle is about $5x10^{-9}$ cm^2/sec (from fig 2-11). The first-order decay coefficient for the large particle is therefore:

$$\omega_f / D = \frac{6.7x10^{-5} cm/\sec}{5x10^{-9} cm^2/\sec} = 1.3x10^4/cm$$

The distance at which the concentration is halved is:

$$\frac{1}{2} = e^{-(1.3x10^4/cm)\bullet x}$$

x = $5x10^{-5}$ cm or 0.5 μm

Example 2-4

What is the minimum distance the particles will travel before settling to the river bottom? Assume a 2-m river depth and 200 μm particles, 2.6 g/cm^3 particle density and $1.3x10^{-2}$ cm^2/sec kinematic viscosity.

Settling rate:

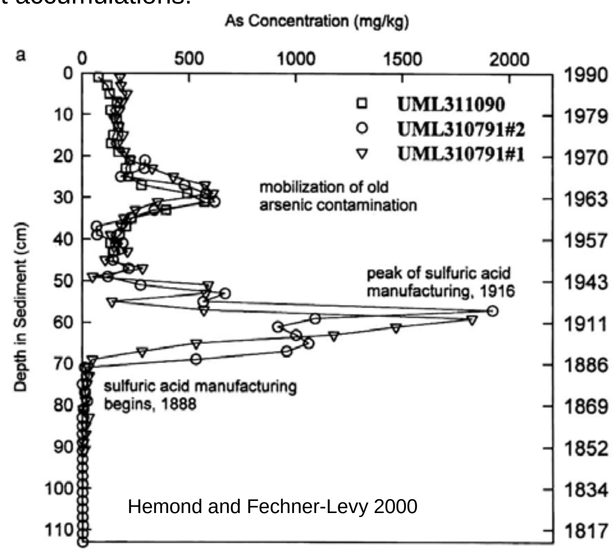
$$\omega_f = \frac{(2/9)\bullet 981 cm/\sec^2 \bullet ((2.6g/cm^3)/(1g/cm^3)-1)\bullet(10^{-2}cm)^2}{0.013cm^2/\sec}$$

$$\omega_f = 2.7cm/\sec$$

The time required to settle 2 m is therefore 75 sec, and the particle will travel:

$$75\sec \bullet 0.2m/\sec = 15m$$

Sediment accumulations:



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Example 2-5

Sediment from a 10 cm depth of a lake has a ^{210}Pb activity of 2.5 disintegrations per minute (DPM), while surface sediment has a DPM of 4. How rapidly does sediment accumulate in this lake?

Using the basic equation for radioactive decay:

$$t = \frac{-1}{\lambda}\bullet \ln\left(\frac{A_d}{A_o}\right) = \frac{-1}{0.03/year}\ln\left(\frac{2.5DPM}{4DPM}\right) = 16/year$$

$$\frac{10cm}{16year} = 0.6cm/year$$